

G.17

```
open Sec. 3nCotesRule:=  
n->fac for(int(interp([seq(i*h,i=1..n-1)],  
[seq(f(i*h),i=1..n-1)],z),z=0..n*h);
```

```
x=-1:2/n:1;  
[X,Y]=meshgrid(x,x);  
Z=X+1*Y;  
for i=1:m  
  Z=Z-(Z.^3-1)/(3*Z^2);  
end;
```

TEXTS IN COMPUTATIONAL SCIENCE
AND ENGINEERING

11

Walter Gander · Martin J. Gander · Felix Kwok

Scientific Computing

An Introduction
using Maple and MATLAB

Editorial Board

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